ON MANY-VALUED FUZZY ROUGH SETS GENERATED BY FAMILIES OF \(L\)-RELATIONS

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In the second part of the last century the mankind has met a new problem – to "digest" the avalanche-like increase of information. Moreover, the solution of this problem was additionally complicated since more and more frequently scientists and practitioners had to deal with non-precise, so called "vague" information. This aroused the necessity to invent qualitatively new mathematical tools in order to manage the problem. Responding to this challenge, three alternative approaches and respective mathematical theories were worked out: fuzzy logic and fuzzy sets initiated by L.A. Zadeh [6], soft sets initiated by Molodtsov and rough sets initiated by Z. Pawlak [4]. Although these theories originally started from different points of view, were based on different assumptions, and had different merits, soon there appeared successful "hybrids" obtained by synthesis of these approaches. As the first and, in our opinion, the most prospective one is the theory of fuzzy rough and rough fuzzy sets [1]. Just in this direction is our research to be presented at the conference.

The context for our research is determined by sets \(X\) equipped with \(L\)-relations \(\rho : X \times X \rightarrow L\) where \(L\) is a lattice enriched with a \(t\)-norm, that is a commutative associative binary operation \(* : L \times L \rightarrow L\) (see e.g. [3]). In case \(L = [0, 1]\) the most important examples of \(t\)-norms are the minimum \(t\)-norm \(\alpha \land \beta = \min(\alpha, \beta)\), the product \(t\)-norm \(\alpha \odot \beta = \alpha \cdot \beta\) and Lukasiewicz \(t\)-norm \(\alpha \ast L \beta = \max\{1 - \alpha + \beta, 0\}\). An \(L\)-relation is called an equivalence \(L\)-relation if it is reflexive \((\rho(\alpha, \alpha) = 1)\), symmetric \((\rho(\alpha, \beta) = \rho(\beta, \alpha))\) and transitive \((\rho(\alpha, \beta) \leq \rho(\alpha, \gamma) \ast \rho(\gamma, \beta))\) for all \(\alpha, \beta, \gamma \in L\). Proceeding with our research which was initiated in [2], we study \(L\)-rough sets generated by equivalence \(L\)-relations on a given set. However, as different from the situation considered in [2], now our interest is in lattice-ordered systems of \(L\)-rough sets which are generated by families \(\{\rho_{\lambda} \mid \lambda \in M\}\) of \(L\)-relations where \(M\) is a complete bounded lattice (e.g. \(M = \{0, 1\}\) or \(M = [0, 1]\)) and \(\rho_{\lambda_1} \leq \rho_{\lambda_2}\) whenever \(\lambda_1 \leq \lambda_2\). In the result we obtain an \(M\)-graded \(L\)-rough sets. We study \(M\)-graded \(L\)-rough sets in the framework of the category of \(M\)-approximate systems, see [5]. In particular, we are interested in the study of the lattice properties of families of \(M\)-graded \(L\)-rough sets, their categorical properties, the relations of \(M\)-graded \(L\)-rough sets with different approaches to fuzzy topology. We plan to discuss also some possible applications of \(M\)-graded \(L\)-rough sets.

REFERENCES