QUADRATIC/LINEAR RATIONAL SPLINE COLLOCATION FOR BOUNDARY VALUE PROBLEMS

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We consider the boundary value problem

\[ y''(x) + p(x)y'(x) + q(x)y(x) = f(x), \quad x \in (a, b), \]
\[ y(a) = \alpha, \quad y(b) = \beta, \]

and suppose it has the unique solution.

We will discuss the quadratic/linear rational spline collocation on the uniform mesh with points \( x_i = a + ih, \ i = 0, \ldots, n, \ h = (b - a)/n. \) The quadratic/linear rational spline \( S \) is assumed to be of class \( C^2 \) and having the form

\[ S(x) = a_i + b_i(x - x_{i-1}) + \frac{c_i(x - x_{i-1})}{1 + d_i(x - x_{i-1})}, \quad x \in [x_{i-1}, x_i], \quad 1 + d_i(x - x_{i-1}) > 0, \quad i = 1, \ldots, n. \]

The collocation method leads to a nonlinear system to determine the spline coefficients \( a_i, b_i, c_i, d_i. \)

Quadratic/linear rational spline is always convex (or concave). Therefore, it is a reasonable approximate solution only if the exact solution of the problem has same properties.

We show that for the quadratic/linear rational spline \( S \) with the solution \( y \) of the boundary value problem which is a strictly convex (or strictly concave) function it holds \( \|S - y\|_\infty = O(h^2). \) We prove also convergence rates \( \|S' - y'\|_\infty = O(h^2) \) and \( \|S'' - y''\|_\infty = O(h^2). \)

We compare the obtained results with cubic spline collocation method ones. The convergence rate \( O(h^2) \) with adequate error estimate for cubic spline collocation method is known [1].

We also give corresponding numerical examples.

REFERENCES