NUMERICAL ACCURACY OF A FAMILY OF PARAMETER CHOICE RULES IN TIKHONOV METHOD FOR SOLVING ILL-POSED PROBLEMS WITH NOISY OPERATOR

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We consider a linear ill-posed problem $A_\ast x = y_\ast$ with $A_\ast \in \mathcal{L}(X,Y)$, where $X$ and $Y$ are Hilbert spaces. We assume that instead of exact $A_\ast$ and $y_\ast$ only their approximations $A \in \mathcal{L}(X,Y)$ and $y \in Y$ are available with $\|A - A_\ast\| \approx \eta$ and $\|y - y_\ast\| \approx \delta$. To solve the problem $Ax = y$, we use the Tikhonov method $x_\alpha = (\alpha I + A^*A)^{-1}A^*y$, where $\alpha$ is the regularization parameter.

In [1] a general family of parameter choice rules was proposed for the exact operator. This family includes many known rules as special cases and guarantees convergence of approximations. Many rules from this family work well also in the case of large under- or overestimation of the noise level of the right hand side. In [2], rules of the type $d(\alpha) = b(\delta + \eta \|x_\alpha\|)$ were investigated theoretically with regard to the quasioptimality property. In this talk we compare the numerical accuracy of these rules, where $d(\alpha)$ is the function that determines the abovementioned general family.

The results show that in this family there still exist rules that are stable with respect to moderate underestimation of the noise level $\delta$, regardless whether $\eta$ is large or small. If $\eta$ is much larger than $\delta$, then the sensitivity of rules to the underestimation of $\delta$ is generally less (since then $\|x_\alpha\|$ typically grows). If $\eta$ is much smaller than $\delta$, then there are fewer rules that work well in the case of underestimation of $\delta$ (then $\delta$ dominates in $\delta + \eta \|x_\alpha\|$).

REFERENCES