Abstracts of MMA2016, June 1–4, 2016, Tartu, Estonia

ORDER REDUCTION PHENOMENON FOR GENERAL LINEAR METHODS

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To investigate a possible order reduction for general linear methods [2] we consider the Prothero-Robinson test problem [3]

$$\begin{cases} y'(t) = \lambda\big(y(t) - \varphi(t)\big) + \varphi'(t), \quad t \in [t_0, T], \\ y(t_0) = \varphi(t_0), \end{cases}$$

$\mathrm{Re}(\lambda) \le 0$, with exact solution $y(t) = \varphi(t)$.

Let h be a stepsize and $z = h\lambda$. We are interested in the behavior of the global error $e^{[n]}$ at n-th step as $h \to 0$ and $z = O(h)$, which corresponds to the classical non-stiff case, and as $h \to 0$ and $z \to -\infty$, which corresponds to the stiff case, i.e., when $\mathrm{Re}(\lambda) \ll 0$, compare [1]. It can be demonstrated that in the non-stiff case we have $\|e^{[n]}\| = O(h^p)$ as $h \to 0$, if $\|e^{[0]}\| = O(h^p)$ and the general linear method has order p, regardless of the stage order q.

In the stiff case we assume that the general linear method has order p and stage order $q \le p$ and $\|e^{[0]}\| = O(h^p)$ as $h \to 0$. Then

$$\|e^{[n]}\| = O(h^q) + O(h^p) \quad \text{as} \quad h \to 0 \quad \text{and} \quad z \to -\infty.$$

The global error estimate can be improved for general linear methods with so-called Runge–Kutta stability which are A-stable:

$$\|e^{[n]}\| = O(h^{q+1}) + O(h^p) \quad \text{as} \quad h \to 0 \quad \text{and} \quad z \to -\infty.$$

Theoretical results are illustrated by examples of methods and numerical experiments.

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