

ON THE ACCURACY OF SOME ABSORBING BOUNDARY CONDITIONS FOR THE SCHRÖDINGER EQUATION

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We consider the linear one-dimensional Schrödinger equation

$$\begin{cases} \frac{\partial u}{\partial t} = i \frac{\partial^2 u}{\partial x^2}, t \in [0, T], x \in \Omega, \\ u|_{t=0} = u_0(x). \end{cases} \quad (1)$$

The aim of this research is to investigate the accuracy of different absorbing boundary conditions. In practice to solve (1) often artificial boundary conditions are used. However, if the standard boundary conditions are used (lets say Dirichlet boundary conditions) the wave (modelled by equation) after reaching such boundary will be mirrored back and will corrupt the results of all further modelling. There exist different approaches to solve this problem.

Straightforward way to avoid the corruption of results is to increase the modelling interval Ω to delay the corruption of results by delaying the reflection of wave, since it lets waves to travel in one direction for a longer time interval. However, for a long term modelling this strategy is highly inefficient. For this reason different absorbing boundary conditions were developed. These boundary conditions absorb the waves as they reach the boundary area.

In [2] it is shown how to derive several versions of exact transparent boundary conditions. However, often these type of conditions requires to store and process a big amount of data from the past of the solution, so it may be inefficient.

We will focus on the boundary conditions that approximate the boundary accurately enough but still with some amount of computational error. J. Szeftel in [1] proposed a technique to construct such boundary conditions. He introduced the reflection coefficient and used it as a goal functional in minimization problem, which is solved by the simplex method. In our work we analyze the optimality of the proposed technique by performing alternative fitting of parameters for the proposed example.

At the second stage of our research, these absorbing boundary conditions will be tested for solving nonlinear Schrödinger problems.

REFERENCES

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