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NUMERICAL METHODS FOR GENERAL VOLTERRA INTEGRAL EQUATIONS OF LIGHTHILL-TYPE

TERESA DIOGO

CEMAT/Instituto Superior Técnico, University of Lisbon

Av. Rovisco Pais,1, 1049-001 Lisboa, Portugal

E-mail: `tdiogo@math.ist.utl.pt`

This talk is concerned with the construction and analysis of numerical methods for a general class of nonlinear second-kind Volterra integral equations

$$y(z) = f(z) - \int_0^z \frac{x^\beta}{(z-x)^\alpha} g(y(x))dx, \quad 0 < \alpha < 1,\ \beta > 0, \quad z \in [0,T], \tag{1}$$

which have important applications in nonlinear problems of heat conduction, boundary-layer heat transfer, chemical kinetics and theory of superfluidity. The above class of equations has been considered by the authors in [2]. The kernel of these equations, $x^\beta (z-x)^{-\alpha} g(y(x))$, with $\alpha \in (0,1)$ and $\beta > 0$, may possess two types of singularities, their solutions being typically nonsmooth.

In previous works a particular case of (1), with $\alpha = 2/3, \beta = 1/3,\ f(z) = 1$ and $g(y) = y^4$ (Lighthill's equation), was considered. The derivative of its solution behaves like $y'(t) \sim t^{-1/3}$, near the origin. As it would be expected, the typical nonsmooth properties of y cause a drop in the global convergence orders of numerical methods based on uniform meshes like collocation and product integration methods. Some classical techniques can be used to recover the optimal convergence orders and this was done for Lighthill's equation. We refer to [3], where a collocation method with graded meshes was proposed, and to [4] where the basis for the approximating space also includes some fractional powers.

Recently, in [2], a new approach has been used for the solution of the general equation (1), where an initial integral over a small interval is calculated analytically, by using a series solution available near the origin; this, combined with a product integration-type method, leads to optimal convergence rates. On the other hand, a Jacobi spectral collocation method has been proposed in [1], where a transformation of the independent variable is first introduced in order to obtain a new equation with a smoother solution. The Jacobi collocation method is then applied to the transformed equation and a complete convergence analysis of the method is carried out for the L^∞ and the L^2 norms. The exponential decay of the errors is well illustrated with numerical examples.

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