

## A NUMERICAL METHOD FOR MIXED BOUNDARY VALUE PROBLEMS ON DOMAINS WITH CORNERS

LUISA FERMO and CONCETTA LAURITA

*Department of Mathematics and Computer Science, University of Cagliari*

Viale Merello 92, 09123 Cagliari, Italy

E-mail: [fermo@unica.it](mailto:fermo@unica.it)

*Department of Mathematics, Computer Science and Economics, University of Basilicata*

Via dell'Ateneo Lucano 10, 85100 Potenza, Italy

E-mail: [concetta.laurita@unibas.it](mailto:concetta.laurita@unibas.it)

This talk deals with the numerical solution of mixed Dirichlet-Neumann boundary value problems for the Laplace equation

$$\begin{cases} \Delta u(P) = 0, & P \in D, \\ u(P) = f_D(P), & P \in \Sigma_D, \\ \frac{\partial u(P)}{\partial n_P} = f_N(P), & P \in \Sigma_N, \end{cases}$$

where  $D$  is a simply connected bounded region in the plane with a piecewise smooth-boundary  $\Sigma = \Sigma_D \cup \Sigma_N$ ,  $f_D$  and  $f_N$  are given functions on  $\Sigma_D$  and  $\Sigma_N$ , respectively and  $n_P$  is the inner normal vector to  $\Sigma_N$  at  $P$ .

Following a well-known theory, we represent the solution  $u$  as the single layer potential

$$u(A) = \int_{\Sigma} \Psi(Q) \log |A - Q| d\Sigma_Q, \quad A \in D,$$

where  $\Psi$  denotes the unknown single layer density function satisfying the following system

$$\begin{cases} \int_{\Sigma} \Psi(Q) \log |P - Q| d\Sigma_Q = f_D(P), & P \in \Sigma_D, \\ \pi \Psi(P) + \int_{\Sigma} \Psi(Q) \frac{\partial}{\partial n_P} \log |P - Q| d\Sigma_Q = f_N(P), & P \in \Sigma_N. \end{cases}$$

The latter is characterized by a boundary integral equation of the second kind having Mellin-type integral operators. Hence, for its numerical solution, we propose a method of Nyström type based on Legendre quadrature formulas which are “modified” around the corners.

We examine the stability of the proposed method, discuss the convergence, which is accelerated by using regularized technique, and, finally, illustrate numerical tests showing the efficiency of the new approach.