

ISOSPECTRAL DOMAINS FOR DISCRETE ELLIPTIC OPERATORS

LORELLA FATONE¹ and DANIELE FUNARO²

¹ *Dipartimento di Matematica e Informatica*

Università di Camerino, Via Madonna delle Carceri 9, 62032 Camerino (Italy)

E-mail: lorella.fatone@unicam.it

² *Dipartimento di Fisica, Informatica e Matematica*

Università di Modena e Reggio Emilia, Via Campi 213/B, 41125 Modena (Italy)

E-mail: daniele.funaro@unimore.it

We consider the eigenvalues of the Laplace operator in 2-D with homogeneous Dirichlet boundary conditions (see, e.g., [1] for a general overview). It is known that there are distinct domains (non isometric) such that all the infinite eigenvalues coincide. For this reason, these are called isospectral domains. It is not known however if it is possible to connect with continuity two isospectral domains through a sequence of domains, by preserving the whole spectrum. Partial answers can be given by working in a finite dimensional environment. Here, we take a suitable approximation of the Laplace operator corresponding to a negative-definite matrix. By varying the domain, we are interested to detect those deformations that preserve the entire set of eigenvalues (that are now in finite number). At the same time, not all the possible deformations are allowed, but only those belonging to a finite dimensional space of parameters. In particular, the question examined here concerns with the deformation of quadrilateral domains, with the aim of preserving the eigenvalues of discrete operators obtained from collocation of the Laplace problem using polynomials of degree 3 in each variable. The vertices of the domains are then suitably moved by maintaining the magnitude of the corresponding eigenvalues. The results show that, at least in these simplified circumstances, families of isospectral domains exist and can be connected by continuous transformations. The most straightforward (but extremely expensive) approach is to try all the possible allowed configurations on a fine grid, and sorting the isospectral ones by comparing the corresponding spectrum. An upgraded version is to move the vertices along curves, whose tangent is obtained as an application of the inverse function theorem.

REFERENCES

- [1] D. S. Grebenkov, B.-T. Nguyen, Geometrical Structure of Laplacian Eigenfuncions, arXiv:1206.1278, 2012.