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ON THE SELF-REGULARIZATION OF INTEGRAL EQUATIONS OF THE FIRST KIND BY THE SPECIAL COLLOCATION METHOD

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We consider the integral equation of the first kind

$$(A_0u)(t) \equiv \int_0^1 K_0(t,s)\,x(s)\,\mathrm{d}s = y_0(t) \quad (0 \le t \le 1), \tag{1}$$

where $A_0: \ L_2(0,1) \to L_2(0,1)$, $\mathcal{N}(A_0) = \{0\}$ and $y_0 \in C[0,1]$.

We assume that instead of the function $y_0(t)$ and the kernel $K_0(t,s)$ the approximations $y(t_i)$, $K(t_i,s)$ are known on knot set $\{t_i \in [0,1],\ i \in I,\ t_i \ne t_j\ (i \ne j)\}$ with given error levels δ_i, h_i:

$$\begin{gathered}\big|y(t_i) - y_0(t_i)\big| \le \delta_i \quad (i \in I),\\ \max\nolimits_{0\le s\le 1} |K(t_i,s) - K_0(t_i,s)| \le h_i \quad (i \in I).\end{gathered}$$

Assume that $\{K(t_i,s), i \in I\}$ is a linearly independent system in the space $L_2(0,1)$ for all $s \in [0,1]$. For approximate solution of the equation (1) we use the special collocation method [2, 5], where approximate solution has form $x_n(s) = \sum_{i\in I_n} c_{n,i}K(t_i,s)$ and the coefficients $\{c_{n,i}\}$ are found from the system

$$\sum_{i\in I_n} c_{n,i} \int_0^1 K(t_i,s)\,K(t_j,s)\,\mathrm{d}s = y(t_j) \quad (j \in I_n).$$

We give the monotone error rule for the choice of the discretization level as the index $n = n_{ME}$ of expanding index sets $I_n \subset I_{n+1} \subset I, \quad n \in N$ of the sets of collocation points $\{t_i, i \in I_n\}$, guaranteeing monotone decrease of error $\|x_n - x_*\| \le \|x_{n-1} - x_*\| (n \le n_{ME})$. Note that the monotone error rule in other regularization and self-regularization methods is considered in [1–4].

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