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χ-UNIVERSALITY OF TWISTS OF ELLIPTIC CURVES

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Let E be an elliptic curve over the field of rational numbers with discriminant $\Delta \neq 0$. For a prime p, denote by E_p the reduction modulo p of E over $\mathbb{F}_p$, and define $\lambda(p)$ by the equality $|E(\mathbb{F}_p)| = p + 1 - \lambda(p)$, where $|E(\mathbb{F}_p)|$ is the number of points of E_p. Denote by $L_E(s)$, $s = \sigma + it$, the L-function of the curve E.

Now let χ be a Dirichlet character modulo a prime q. For $\sigma > \frac{3}{2}$, the twist $L_E(s, \chi)$ of $L_E(s)$ is defined by

$$L_E(s,\chi) = \prod_{p|\Delta} \left(1 - \frac{\lambda(p)\chi(p)}{p^s}\right)^{-1} \prod_{p\nmid\Delta} \left(1 - \frac{\lambda(p)\chi(p)}{p^s} + \frac{\chi^2(p)}{p^{2s-1}}\right)^{-1},$$

and is analytically continued to an entire function. We consider the universality of $L_E(s, \chi)$ with respect to characters χ. Let, for $Q \geq 2$,

$$M_Q = \sum_{q \leq Q} \sum_{\substack{\chi=\chi(\mathrm{mod} q) \\ \chi\neq\chi_0}} 1, \quad \text{and} \quad \mu_Q(...) = \frac{1}{M_Q} \sum_{q \leq Q} \sum_{\substack{\chi=\chi(\mathrm{mod} q) \\ \chi\neq\chi_0 \\ ...}} 1,$$

where χ_0 is the principal character modulo q, and, in place of dots, we will write a condition satisfied by the pair $(q, \chi(\mathrm{mod} q))$. Moreover, let $\mathcal{K}$ be the class of compact subsets of the strip $\{s \in \mathbb{C} : 1 < \sigma < 3/2\}$ with connected complements, and $H_0(K)$, $K \in \mathcal{K}$, be the class of continuous non-vanishing functions on K which are analytic in the interior of K. The main result of the report is the following theorem.

Theorem 1. *Suppose that $K \in \mathcal{K}$ and $f(s) \in H_0(K)$. Then, for every $\varepsilon > 0$,*

$$\liminf_{Q\to\infty} \mu_Q \left\{ \sup_{s\in K} |L(s,\chi) - f(s)| < \varepsilon \right\} > 0.$$

For the proof, a limit theorem obtained in [1] is applied.

REFERENCES

[1] A. Laurinčikas. An Elliott-type theorem for twists of L-functions of elliptic curves. *Math. Notes*, **99** (1):82–90, 2016.