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ON COMPARISON OF ACCURACY OF APPROXIMATE SOLUTIONS OF OPERATOR EQUATIONS [1]

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Let $A\colon X \to Y$ be a linear bounded operator between Hilbert spaces. We consider the equation

$$Ax = y, \qquad y \in \mathcal{R}(A), \tag{1}$$

where noisy data y^δ are given and we have the noise level information in the form

$$\|D(y^\delta - y)\| \le \delta,$$

where D is a linear injective, possibly unbounded operator in Y with domain $\mathcal{D}(D)$. We assume that $y^\delta, y \in \mathcal{D}(D)$. Let x_* be a solution of the problem (1). The following simple result [2] enables to compare the accuracy of different approximate solutions of problem $Ax = y$.

THEOREM 1. *Let $x, x' \in X$, $x' = x + A^*z, z \in Y \cap \mathcal{D}((D^{-1})^*)$ and $w = x + A^*z/2$. Then it holds the implication*

$$d(z) := \frac{(y^\delta - Aw, z)}{\|(D^{-1})^*z\|} > \delta \Longrightarrow \|x' - x_*\| < \|x - x_*\|,$$

$$\frac{1}{2}(\|x' - x_*\|^2 - \|x - x_*\|^2) = (Aw - y, z) \le \phi(z) := \frac{1}{2}\|A^*z\|^2 + (Ax - y^\delta, z) + \delta\|(D^{-1})^*z\|. \tag{2}$$

This theorem generalizes some previous results about monotonicity of error of approximate solutions generated by the same method but using different parameters (see [1, 3, 4]).

If element $x \in X$ is fixed, one may consider the problem of minimizing the function $\phi(z)$ in (2).

Proposition 1. The function $\phi(z)$ is minimized by the solution $z \in Y \cap \mathcal{D}(D^{-1}(D^{-1})^*)$ of the equation $AA^*z + \delta D^{-1}(D^{-1})^*z/\|(D^{-1})^*z\| = y^\delta - Ax$.

Proposition 2. Let z_α be the solution of the equation $\alpha z_\alpha + \frac{1}{2}AA^*z_\alpha = y^\delta - Ax$, where $\alpha \in R$, $\alpha > 0$. Then it holds the implication $\alpha\|z_\alpha\| > \delta \Longrightarrow \|x + A^*z_\alpha - x_*\| < \|x - x_*\|$.

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