

DIFFERENCE SCHEME FOR TWO-DIMENSIONAL PARABOLIC EQUATION WITH AN INTEGRAL BOUNDARY CONDITION ¹

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We consider linear two-dimensional parabolic equation with integral boundary condition

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left(k(x, y) \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(k(x, y) \frac{\partial u}{\partial y} \right) - c(x, y)u + f(x, y, t), \quad (x, y) \in \Omega, \quad (1)$$

$$u(x, y, t) = \iint_{\Omega} K(x, y, \xi, \eta) d\xi d\eta + \mu(x, y, t), \quad (x, y) \in \partial\Omega, \quad (2)$$

$$u(x, y, 0) = \varphi(x, y), \quad (x, y) \in \Omega, \quad (3)$$

where $\Omega = \{0 < x, y < 1\}$. This problem is solved using the semi-implicit difference scheme:

$$\frac{u_{ij}^{n+1} - u_{ij}^n}{\tau} = \Lambda u_{ij}^{n+1} + f_{ij}^{n+1}, \quad i, j = 1, 2, \dots, N - 1, \quad (4)$$

$$u_{ij}^{n+1} = h^2 \sum_{k,l=0}^N \rho_{kl} K_{kl}(x_i, y_j) u_{kl}^n + \mu_{ij}^n, \quad (i, j) = \{(0, j), (N, j), (i, 0), (i, N)\}, \quad (5)$$

$$u_{ij}^0 = \varphi_{ij}, \quad i, j = 0, 1, \dots, N. \quad (6)$$

The principle feature of this scheme is that calculating a double integral by trapezoid rule we take only the previous time layer. The stability of difference scheme is proved by using some properties of M-matrices. For solution of the system of difference equations (4)-(6) we use the alternating direction method, and other methods.

REFERENCES

- [1] Y. Lin, S. Xu, H. M. Yin. Finite difference approximations for a class of non-local parabolic equations. *Intern. J. Math. and Mat. Sci.*, **20** (1):147–164, 1997.
 [2] K. Jakubėlienė and M. Sapagovas. On the stability of a difference scheme for a two-dimensional parabolic equation with an integral condition. *Lith. Math. J.*, **53** (3):311–323, 2013.

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