

## THE LOG-EXTENDED EXPONENTIAL-GEOMETRIC DISTRIBUTION AND AN ASSOCIATED REGRESSION MODEL

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The development of new parametric distributions attracts a great deal of attention with the aim of providing useful probability models in many different areas. In this work, a new two-parameter continuous probability distribution with bounded support is derived from the extended exponential-geometric distribution introduced in [1]. More specifically, the new random variable,  $X$ , has the following probability density function

$$f(x; \alpha, \beta) = \frac{\alpha(1 + \beta)x^{\alpha-1}}{(1 + \beta x^\alpha)^2}, \quad 0 < x < 1, \quad \alpha > 0, \quad \beta > -1,$$

where  $\alpha$  and  $\beta$  are shape parameters. Some statistical properties of the model are studied. In this regard, we highlight that the moments can be expressed in closed form in terms of the Lerch transcendent function,  $\Phi$ , namely,

$$E[X^k] = 1 - \frac{(1 + \beta)k}{\alpha} \Phi\left(-\beta, 1, 1 + \frac{k}{\alpha}\right), \quad k = 1, 2, \dots$$

Moreover, the proposed distribution has increasing generalized failure rate, which has fruitful applications in operations management and a tractable expression can be given for the quantile function, which is helpful for the computer-generation of pseudo-random data. The parameter estimation problem is carried out by the method of maximum likelihood (ML), which provides satisfactory results. The Fisher information matrix together with the asymptotic covariance matrix of the ML estimators are also provided. Unified proofs of the above results can be given on the basis of the results in [2]. An application to a real data set illustrates that the new distribution may provide a better fit than other models with bounded domain.

Additionally, a new regression model is introduced by considering the proposed distribution, which is useful for situations where the response variable is restricted to the standard unit interval. The suitability of the regression model is exemplified by means of a real data application.

### REFERENCES

- [1] K. Adamidis, T. Dimitrakopoulou and S. Loukas. On an extension of the exponential-geometric distribution. *Statist. Probab. Lett.*, **73** (3):259–269, 2005.
- [2] P. Jodrá and M.D. Jiménez-Gamero. On a logarithmic integral and the moments of order statistics from the Weibull-geometric and half-logistic families of distributions. *Math. Anal. Appl.*, **410** (2): 882–890, 2014.