

UNIVERSALITY OF SOME COMPOSITE FUNCTIONS RELATED TO PERIODIC ZETA-FUNCTIONS

ANTANAS LAURINČIKAS

Faculty of Mathematics and Informatics, Vilnius University

Naugarduko 24, LT-03225 Vilnius, Lithuania

E-mail: antanas.laurincikas@mif.vu.lt

Let $s = \sigma + it$ be a complex variable, $\mathbf{a} = \{a_m : m \in \mathbb{N}\}$ and $\mathbf{b} = \{b_m : m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}\}$ be two periodic sequences of complex numbers, and $0 < \alpha \leq 1$ be a fixed parameter. The periodic zeta-function $\zeta(s; \mathbf{a})$ and periodic Hurwitz zeta-function $\zeta(s, \alpha; \mathbf{b})$ are defined, for $\sigma > 1$, by the series

$$\zeta(s; \mathbf{a}) = \sum_{m=1}^{\infty} \frac{a_m}{m^s} \quad \text{and} \quad \zeta(s, \alpha; \mathbf{b}) = \sum_{m=0}^{\infty} \frac{b_m}{(m + \alpha)^s},$$

and can be meromorphically continued to the whole complex plane.

In [1], a joint universality theorem on the simultaneous approximation of a given collection of analytic functions by shifts $(\zeta(s + i\tau; \mathbf{a}_1), \dots, \zeta(s + i\tau; \mathbf{a}_{r_1}), \zeta(s + i\tau, \alpha; \mathbf{b}_1), \dots, \zeta(s + i\tau, \alpha; \mathbf{b}_{r_2}))$, $\tau \in \mathbb{R}$, has been obtained. This report is devoted to the universality for composite functions of a collection of the above functions. More precisely, let $H(D)$ be the space of analytic functions on $D = \{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\}$ endowed with the topology of uniform convergence on compacta. We consider the classes of operators $F : H^{r_1+r_2}(D) \rightarrow H(D)$ such that the function

$$F(\zeta(s + i\tau; \mathbf{a}_1), \dots, \zeta(s + i\tau; \mathbf{a}_{r_1}), \zeta(s + i\tau, \alpha; \mathbf{b}_1), \dots, \zeta(s + i\tau, \alpha; \mathbf{b}_{r_2}))$$

is universal in the sense that its shifts approximate a wide class of analytic functions. The results can be found in [2].

REFERENCES

- [1] A. Laurinčikas. Joint universality of zeta-functions with periodic coefficients. *Izv. Math.*, **74** (3):515–539, 2010.
- [2] A. Laurinčikas. Extension of universality for zeta-functions with periodic coefficients. *Sib. Math. J.*, 2016 (to appear).