Abstracts of MMA2016, June 1–4, 2016, Tartu, Estonia

NUMERICAL SIMULATIONS IN TWO-DIMENSIONAL NEURAL FIELDS

PEDRO M. LIMA[1] and EVELYN BUCKWAR[2]

[1] *Centro de Matematica e Aplicacoes, Instituto Superior Tecnico*
Av. Rovisco Pais, 1049-001 Lisboa, Portugal
E-mail: plima@math.ist

[2] *Institute of Stochastics, Johannes Kepler University*
Altenberger Strasse 69, 4040 Linz, Austria
E-mail: evelyn.buckwar@jku.at

The purpose of the present work is to describe a new efficient numerical method for Neural Field Equations (NFE), targeted directly for the application in a Computational Neuroscience and Cognitive Robotics context. We consider the NFE in the form

$$c\frac{\partial}{\partial t}V(\bar{x},t) = I(\bar{x},t) - V(\bar{x},t) + \int_\Omega K(|\bar{x}-\bar{y}|)S(V(y,t-\tau(\bar{x},\bar{y}))d\bar{y}, \quad t \in [0,T], \quad \bar{x} \in \Omega \subset \mathbb{R}^2, \tag{1}$$

where the unknown $V(\bar{x},t)$ is a continuous function $V : \Omega \times [0,T] \to \mathbb{R}$, I, K and S are given functions; c is a constant. We search for a solution V of this equation which satisfies the initial condition

$$V(\bar{x},t) = V_0(\bar{x},t), \qquad \bar{x} \in \Omega, \quad t \in [-\tau_{max}, 0], \tag{2}$$

where $\tau_{max} = \max_{\bar{x},\bar{y}\in\Omega} \tau(\bar{x},\bar{y})$. Here τ is a delay depending on $\bar{x}$ and $\bar{y}$ (as a particular case, we also consider the case $\tau \equiv 0$).

Equation (1) without delay was introduced first by Wilson and Cowan [3], and then by Amari [1], to describe excitatory and inhibitory interactions in populations of neurons. $V(\bar{x},t)$ represents the post-synaptic neuronal membrane potential at instant t and position $\bar{x}$. The function I represents external sources of excitation and S describes the dependence between the firing rate of the neurons and their membrane potential (typically, it is a function of sigmoidal type). The kernel function $K(|\bar{x}-\bar{y}|)$ gives the connectivity between neurons in positions $\bar{x}$ and $\bar{y}$. The delay $\tau(\bar{x},\bar{y})$ takes into consideration the time spent by an electrical signal to travel between positions $\bar{x}$ and $\bar{y}$.

We introduce an implicit scheme with second order discretization in time.

In the two-dimensional case, the required computational effort to solve equations (1) grows very fast as the discretization stepsize in space is reduced, and therefore special attention has to be paid to the creation of effective methods. In our work, we use Gaussian quadratures for the integration in space, and combine this with low-rank methods, which enable a significant reduction of the dimensions of the matrices, without affecting the final accuracy of the method.

In the present talk we will present some numerical results obtained by the application of the present algorithm to the analysis of some neural fields arising in cognitive robotics (see [2]).

REFERENCES

[1] S.L. Amari. Dynamics of pattern formation in lateral-inhibition type neural fields. *Biol. Cybernet.*, **27** (2):77–87, 1977.

[2] W. Erlhagen and E. Bicho. The dynamic neural field approach to cognitive robotics. Journal of Neural Engineering, **3** (3): 36–54, 2006.

[3] H.R. Wilson and J.D. Cowan. Excitatory and inhibitory interactions in localized populations of model neurons. Bipophys. J., **12** 1–24, 1972.