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ON JOINT UNIVERSALITY OF DIRICHLET *L*-FUNCTIONS

ANTANAS LAURINČIKAS[1], **RENATA MACAITIENĖ**[2] and DARIUS ŠIAUČIŪNAS[2]

[1]*Faculty of Mathematics and Informatics, Vilnius University*

Naugarduko 24, LT-03225 Vilnius, Lithuania

E-mail: `antanas.laurincikas@mif.vu.lt`

[2]*Faculty of Technology, Physical and Biomedical Sciences, Šiauliai University*

P. Višinskio 19, LT-77156 Šiauliai, Lithuania

E-mail: `renata.macaitiene@mi.su.lt, darius.siauciunas@su.lt`

Let χ be a Dirichlet character, and $L(s,\chi)$, $s=\sigma+it$, denote the corresponding Dirichlet L-function defined, for $\sigma>1$, by the series $L(s,\chi)=\sum_{m=1}^{\infty}\frac{\chi(m)}{m^s}$, and by analytic continuation elsewhere. It is well known that each function $L(s,\chi)$ is universal, and also a collection of Dirichlet L-functions with pairwise non-equivalent characters are jointly universal.

Our report is devoted to joint discrete universality of Dirichlet L-functions. Let $\mathcal{K}$ be the class of compact subsets of the strip $\{s\in\mathbb{C}:\ \frac{1}{2}<\sigma<1\}$ with connected complements, and $H_0(K)$, $K\in\mathcal{K}$, be the class of continuous non-vanishing functions on K which are analytic in the interior of K. Denote by $\mathbb{P}$ the set of all prime numbers, and, for $h_j>0$, $j=1,\dots,r$, define the set $L(h_1,\dots,h_r;\pi)=\{(h_1\log p:\ p\in\mathbb{P}),\dots,(h_r\log p:\ p\in\mathbb{P});\pi\}$. Then in [1] the following theorem has been proved.

THEOREM 1. *Suppose that $\chi_1,\dots,\chi_r$ are pairwise non-equivalent Dirichlet characters, and that the set $L(h_1,\ \dots,h_r;\pi)$ is linearly independent over the field of rational numbers $\mathbb{Q}$. For $j=1,\dots,r$, let $K_j\in\mathcal{K}$ and $f_j(s)\in H_0(K_j)$. Then, for every $\varepsilon>0$,*

$$\liminf_{N\to\infty}\frac{1}{N+1}\#\left\{0\le k\le N:\ \sup_{1\le j\le r}\sup_{s\in K_j}|L(s+ikh_j,\chi_j)-f_j(s)|<\varepsilon\right\}>0.$$

We replace the shifts $L(s+ikh_j,\chi_j)$ in Theorem 1 by more general ones. Let $L(h_1,\dots,h_r)=\{(h_j\log p:\ p\in\mathbb{P}),\ j=1,\dots,r\}$.

THEOREM 2. *Suppose that $\chi_1,\dots,\chi_r$ are pairwise non-equivalent Dirichlet characters, $\alpha\in(0,1)$ is a fixed number, and the set $L(h_1,\dots,h_r)$ is linearly independent over $\mathbb{Q}$. For $j=1,\dots,r$, let $K_j\in\mathcal{K}$ and $f_j(s)\in H_0(K_j)$. Then, for every $\varepsilon>0$,*

$$\liminf_{N\to\infty}\frac{1}{N+1}\#\left\{0\le k\le N:\ \sup_{1\le j\le r}\sup_{s\in K_j}|L(s+ik^{\alpha}h_j,\chi_j)-f_j(s)|<\varepsilon\right\}>0.$$

REFERENCES

[1] A. Dubickas and A. Laurinčikas. Joint discrete universality of Dirichlet L-functions. *Arch. Math.*, **104** (1):25–35, 2015.