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TREFFTZ FUNCTIONS IN SOLVING DIRECT AND INVERSE NON-FOURIER PROBLEMS IN A BI-LAYERED COMPOSITE SPHERE

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Non-stationary one-dimensional problems of heat transfer in a bi-layered composite sphere in considered. The layers are made of different substances. Initially, the sphere is maintained at a uniform temperature, and there is a sudden change on its outer surface. The physical and thermal properties of the composite sphere are assumed to be constant. Moreover, no contact thermal resistance between the interface of bi-layered substances appears. The single-phase lag (SPL) model is considered. The governing equations for this problem are

$$\frac{\partial T_i}{\partial t} = -\frac{\alpha_i}{k_i}\frac{1}{r^2}\frac{\partial (r^2 q_i)}{\partial r} = -\frac{\alpha_i}{k_i}\left[\frac{2}{r}q_i + \frac{\partial q_i}{\partial r}\right] \text{ – energy equation,} \tag{1}$$

$$\frac{\partial T_i}{\partial r} = -\frac{1}{k_i}\left[q_i + \tau_i \frac{\partial (q_i)}{\partial t}\right] \text{ – constitutive equation for SPL model,} \tag{2}$$

for the i^{th} layer, $i = 1, 2$, $0 \le r \le r_1$ for the inner layer and $r_1 \le r \le r_2$ for the outer one. Here k_i stands for thermal conductivity and α_i denotes thermal diffusivity for the i^{th} layer; T_i and q_i are temperature and heat flux for the i^{th} layer, respectively. From (1) and (2) the following equation results for temperatures in the i^{th} layer

$$\frac{\partial T}{\partial t} + \tau_i \frac{\partial^2 T}{\partial t^2} = \alpha_i \left(\frac{2}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\right). \tag{3}$$

Equation (3) should be completed by necessary initial and boundary conditions. In case that all the initial and boundary conditions are known, a direct problem is considered. When one of the boundary conditions is not known, a boundary inverse problem appears. In practice, instead of the missing boundary condition, values of the temperature in chosen points inside the body (temperature internal responses) are known.

In order to find an approximate solution of the problem (direct and inverse), the Trefftz functions for the equation (3) need to be found. Then, an approximate solution for the temperature of the i^{th} layer is searched in the form of a linear combination of the Trefftz functions. The heat flux can then be found from the energy equation (1). The Trefftz method enables to solve direct problems as well as inverse ones. In the paper, the Trefftz functions will be derived and appropriate examples will be shown.