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MODIFICATION OF THE MISHOU THEOREM

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Let $s = \sigma + \mathrm{i}t$ be a complex variable, $0 < \alpha \leq 1$ be a fixed parameter, and let, as usual, $\zeta(s)$ and $\zeta(s, \alpha)$ denote the Riemann and Hurwitz zeta-functions, respectively. In [1], H. Mishou obtained a joint universality theorem on the approximation of analytic functions by shifts $\zeta(s + \mathrm{i}\tau)$ and $\zeta(s + \mathrm{i}\tau, \alpha)$, $\tau \in \mathbb{R}$. More precisely, this means that if $\mathcal{K}$ is the class of compact subsets of the strip $\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\}$ with connected complements, and $H_0(K)$ and $H(K)$, $K \in \mathcal{K}$, are the classes of continuous non-vanishing and continuous functions on K, respectively, which are analytic in the interior of K, α is transcendental, and $K_1, K_2 \in \mathcal{K}$, $f_1(K_1) \in H_0(K_1)$ and $f_2(K_2) \in H(K_2)$, then, for every $\varepsilon > 0$,

$$\liminf_{T\to\infty} \frac{1}{T}\mathrm{meas}\left\{\tau \in [0, T] : \sup_{s\in K_1} |\zeta(s + \mathrm{i}\tau) - f_1(s)| < \varepsilon, \sup_{s\in K_2} |\zeta(s + \mathrm{i}\tau, \alpha) - f_2(s)| < \varepsilon\right\} > 0.$$

In the report, we replace "lim inf" in the above inequality by "lim". Let $\mathbb{P}$ be the set of all prime numbers, and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. Then the following statement is valid.

THEOREM 1. *Suppose that the set* $\{(\log p : p \in \mathbb{P}), (\log(m + \alpha) : m \in \mathbb{N}_0)\}$ *is linearly independent over the field of rational numbers. Let* $K_1, K_2 \in \mathcal{K}$ *and* $f_1(s) \in H_0(K_1)$ *and* $f_2(s) \in H(K_2)$. *Then*

$$\lim_{T\to\infty} \frac{1}{T}\mathit{meas}\left\{\tau \in [0, T] : \sup_{s\in K_1} |\zeta(s + \mathrm{i}\tau) - f_1(s)| < \varepsilon, \sup_{s\in K_2} |\zeta(s + \mathrm{i}\tau, \alpha) - f_2(s)| < \varepsilon\right\} > 0$$

holds for all but countably many $\varepsilon > 0$.

A discrete version of Theorem 1 also will be discussed.

REFERENCES

[1] H. Mishou. The joint value-distribution of the Riemann zeta-function and Hurwitz zeta-functions. *Lith. Math. J.*, **47** (1):32–47, 2007.