Abstracts of MMA2016, June 1–4, 2016, Tartu, Estonia
© 2016 Institute of Mathematics and Statistics of the University of Tartu

ON UNIVERSALITY OF THE LERCH ZETA - FUNCTION

ASTA MINCEVIČ

Faculty of Mathematics and Informatics, Vilnius University

Naugarduko str. 24, LT-03225 Vilnius, Lithuania

E-mail: Asta.Mincevic@mif.stud.vu.lt

Let $0 < \alpha \leq 1$ and $\lambda \in \mathbb{R}$ be fixed parameters. The Lerch zeta - function $L(\lambda, \alpha, s)$, $s = \sigma + it$, is defined, for $\sigma > 1$, by the series

$$L(\lambda, \alpha, s) = \sum_{m=0}^{\infty} \frac{e^{2\pi i \lambda m}}{(m+\alpha)^s},$$

and can be analytically continued to the whole complex plane. In [1], an universality theorem on the approximation of a wide class of analytic functions by shifts $L(\lambda, \alpha, s + i\tau)$, $\tau \in \mathbb{R}$, with transcendental α has been obtained. We extend the latter result for a wider class of the parameter α. Let $\mathcal{K}$ be the class of compact subset of the strip $\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\}$ with connected complements, and $H(K)$, $K \in \mathcal{K}$, be the class of continuous functions on K which are analytic in the interior of K. Then we have the following statement.

Theorem 1. *Suppose that the set* $\{\log(m+\alpha) : m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}\}$ *is linearly independent over the field of rational members* $\mathbb{Q}$. *Let* $K \in \mathcal{K}$ *and* $f(s) \in H(K)$. *Then, for every* $\varepsilon > 0$,

$$\liminf_{T\to\infty} \frac{1}{T} \operatorname{meas} \left\{ \tau \in [0, T] : \sup_{s \in K} |L(\lambda, \alpha, s + i\tau) - f(s)| < \varepsilon \right\} > 0.$$

Theorem 1 has a discrete version. Let $h > 0$.

Theorem 2. *Suppose that the set* $\left\{ (\log(m+\alpha) : m \in \mathbb{N}_0), \frac{\pi}{h} \right\}$ *is linearly independent over* $\mathbb{Q}$. *Let* $K \in \mathcal{K}$ *and* $f(s) \in H(K)$. *Then, for every* $\varepsilon > 0$,

$$\liminf_{N\to\infty} \frac{1}{N+1} \# \left\{ 0 \leq k \leq N : \sup_{s \in K} |L(\lambda, \alpha, s + ikh) - f(s)| < \varepsilon \right\} > 0.$$

Here $\operatorname{meas} A$ denotes the Lebesgue measure of a measurable set $A \subset R$, and $\#A$ is the cardinality of A.

REFERENCES

[1] A. Laurincikas. The universality of the Lerch zeta - function. *Lith. Math. J.*, **37** (3):275 280, 1997.