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MOROZOV DISCREPANCY PRINCIPLE FOR ECONOMICAL SOLVING EXPONENTIALLY ILL-POSED PROBLEMS

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The present research is dedicated to the estimates of information complexity for exponentially ill-posed problems. It should be noted that in recent years such problems have been intensively studied under the general theory of the optimal algorithms. Within the framework of this theory, the information complexity is defined as the least amount of discrete information needed to find an approximate solution with given precision. Now we formulate the statement of the problem. Consider a Fredholm equation of the I kind

$$Ax(t) := \int_0^1 a(t,\tau)x(\tau)d\tau = f(t), \qquad t \in [0,1], \tag{1}$$

with the integral operator acting continuously in $L_2 := L_2(0,1)$. Assume that Range(A) is not closed in L_2 and $f \in$ Range(A). We also assume that instead of the right-hand side of (1) a perturbation $f_\delta \in L_2 : \quad \|f - f_\delta\| \le \delta$, $\delta > 0$ is given. The problem (1) is regarded as severely ill-posed if the smoothness of its solution substantially worse than the smoothness of the kernel of A. In this case it is natural to assume that an exact solution $x^\dagger$ satisfies some logarithmic source condition

$$x^\dagger \in M_p(A) := \{u : u = \ln^{-p}(A^*A)^{-1}v, \quad \|v\| \le \rho\}.$$

Here $p, \rho > 0$ are some parameters, A^* is adjoint operator. Such problems are called exponentially ill-posed. Note that the exact information about smoothness of desired solution, namely, the value p is usually not available by practical experiment. For this reason the set

$$M(A) := \bigcup_{p \in (0,p_1)} M_p(A), \tag{2}$$

where $p_1 < \infty$ is an upper bound for possible values p, is considered instead of $M_p(A)$. The aim of the present research is to construct an approximation to the exact solution $x^\dagger$ (1) which has the minimal norm in L_2, and belongs to the set $M(A)$ (2). Here the parameter p is supposed to be unknown.

For the integral equation (1) with finitely smooth kernels some economical projection methods are developed. The ordinary Tikhonov method and a modification of the standard Galerkin scheme which is called hyperbolic cross are used as a regularizer and the discretization scheme, correspondingly. The Morozov discrepancy principle and the balancing principle are applied as stopping rules. It is established that the proposed strategies not only guarantee the optimal oder of accuracy on the class of problems under consideration but also they are economical in the sense of discrete information amount.