

ON THE STABILITY OF DISCRETE NONLOCAL HYPERBOLIC BOUNDARY PROBLEM ¹

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We consider the hyperbolic equation

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = f(x, t), \quad (x, t) \in \Omega \times (0, T],$$

where $\Omega = (0, 1)$, with the classical initial conditions

$$u|_{t=0} = \phi(x), \quad \left. \frac{\partial u}{\partial t} \right|_{t=0} = \psi(x), \quad x \in \bar{\Omega} := [0, 1],$$

and the additional nonlocal integral boundary conditions

$$u(0, t) = \gamma_0 \int_0^1 \beta^0(x) u(x, t) dx + v_l(t), \quad u(1, t) = \gamma_1 \int_0^1 \beta^1(x) u(x, t) dx + v_r(t), \quad t \in [0, T],$$

where $f(x, t)$, $\phi(x)$, $\psi(x)$, $v_l(t)$, and $v_r(t)$ are given functions, γ_0 and γ_1 are given parameters, $\beta^0(x)$ and $\beta^1(x)$ are weight functions. All the coefficients are smooth enough that the solution $U \in C^{4,4}$.

We study the spectrum of the weighted difference operator for the formulated problem [1, 2]. We investigate the spectrum of the finite difference scheme (FDS) and obtain spectral stability conditions subject to boundary parameters γ_0 , γ_1 and piecewise constant weight functions. Obtained stability condition vastly constricts the class of stable weighted FDS.

REFERENCES

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