

CUBIC SPLINE HISTOPOLATION

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For a given histogram with knots $a = x_0 < x_1 < \dots < x_n = b$ and histogram heights z_i , $i = 1, \dots, n$, we choose cubic spline knots $\xi_1 = x_0$, $\xi_i \in (x_{i-1}, x_i)$, $i = 2, \dots, n-1$, $\xi_n = x_n$ and require that the spline S satisfies histopolation conditions

$$\int_{x_{i-1}}^{x_i} S(x)dx = z_i(x_i - x_{i-1}), \quad i = 1, \dots, n.$$

In addition to them we impose two boundary conditions, e.g., $S''(a) = \alpha$, $S''(b) = \beta$, but, instead, the values of S or S' at points a and b could be given as well. Such a spline exists and is unique for any histogram and any choice of spline knots.

We derive linear systems to determine the moments $M_i = S''(\xi_i)$, $i = 1, \dots, n$, and spline parameters

$$\lambda_i = \int_{\xi_i}^{x_i} S(x)dx, \quad \rho_i = \int_{x_i}^{\xi_{i+1}} S(x)dx, \quad i = 1, \dots, n-1,$$

where, in fact, λ_1 and ρ_{n-1} are already known. On each subinterval $[\xi_i, \xi_{i+1}]$, $i = 1, \dots, n-1$, the cubic spline histopolant could be represented via $M_i, M_{i+1}, \lambda_i, \rho_i$.

Another approach to this histopolation problem uses spline values $S_i = S(\xi_i)$, $i = 1, \dots, n$, and allows to use classical representation of cubic spline with the help of spline parameters S_i, M_i .