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GENERALIZED GREEN'S FUNCTIONS TO THE DIFFERENTIAL NONLOCAL PROBLEMS [1]

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We investigate the second order differential problem with nonlocal conditions

$$\mathcal{L}u := a(x)u'' + b(x)u' + c(x)u = f(x), \quad x \in [0,1],$$
$$\langle L_k, u \rangle = 0, \quad k = 1, 2,$$

where $a(x) \neq 0$ for $x \in [0,1]$, $a, b, c \in C[0,1]$, $f \in L^2[0,1]$ are real functions. Let us suppose $\mathbf{L} = (\mathcal{L}, L_1, L_2)^T : H^2[0,1] \to L^2[0,1] \times \mathbb{R}^2$ and $L_1, L_2 \in (H^2[0,1])^*$ are continuous linear functionals.

Considering the differential problem as the operator equation

$$\mathbf{L}u = \mathbf{f}, \qquad \mathbf{f} = (f, 0, 0)^T \in L^2[0,1] \times \mathbb{R}^2,$$

with the domain $D(\mathbf{L}) = H^2[0,1]$, we obtain the existence of the Moore-Penrose inverse $\mathbf{L}^\dagger : L^2[0,1] \times \mathbb{R}^2 \to H^2[0,1]$.

We investigate the minimum norm least squares solution $u^o = \mathbf{L}^\dagger \mathbf{f}$ to the differential problem and present its properties. Moreover, properties of generalized Green's function, representing the minimum norm least squares solution, are also obtained if functionals $L_1, L_2 \in (C^1[0,1])^*$. They resemble the very analogue properties of ordinary Green's function, if it exists [3], and are so similar to the properties of generalized discrete Green's function to the second order discrete problem [2].

REFERENCES

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