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# ABOUT THE USING OF LOCAL MINIMUM POINTS OF THE QUASIOPTIMALITY FUNCTION FOR CHOOSING THE REGULARIZATION PARAMETER

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We consider an operator equation

$$Au = f_*, \quad f_* \in R(A), \tag{1}$$

where $A \in L(H, F)$ is the linear continuous operator between real Hilbert spaces $H$ and $F$. We assume that instead of the exact right-hand side $f_*$ we have only an approximation $f \in F$. To get the regularized solution we consider Tikhonov method $u_\alpha = (\alpha I + A^*A)^{-1}A^*f$, where $\alpha > 0$ is the regularization parameter. If the noise level is unknown then for choosing the regularization parameter some heuristic rule must be used. Let $L_{min}$ be the set of the local minimum points of the quasioptimality criterion function

$$\psi_Q(\alpha) = \alpha \left\| du_\alpha / d\alpha \right\| = \alpha^{-1} \left\| A^*(Au_{2,\alpha} - f) \right\|, \quad u_{2,\alpha} = (\alpha I + A^*A)^{-1}(\alpha u_\alpha - A^*f),$$

on the set of parameters $\Omega = \{\alpha_j : \alpha_j = q\alpha_{j-1},\, j = 1, 2, ..., M,\, 0 < q < 1\}$. Then for the local minimum points of the function $\psi_Q(\alpha)$ the following error estimates hold:

a)

$$\min_{\alpha \in L_{min}} \|u_\alpha - u_*\| \leq C \min_{\alpha_M \leq \alpha \leq \alpha_0} \left\{ \left\| u_\alpha^+ - u_* \right\| + \left\| u_\alpha - u_\alpha^+ \right\| \right\}.$$

Here $u_*$ is the solution of the problem (1), $u_\alpha^+$ is the approximate solution with exact right-hand side $f_*$ and the constant $C \leq c_q \ln(\alpha_0/\alpha_M)$ can be computed for each individual problem $Au = f$.

b) Let $u_* = (A^*A)^{p/2}v$, $\|v\| \leq \rho$. If $\alpha_0 = 1$, $\alpha_M = c\left\| f - f_* \right\|^2$, $c = (2\left\| u_* \right\|)^{-2}$, then

$$\min_{\alpha \in L_{min}} \|u_\alpha - u_*\| \leq c_{p,q} \rho^{1/(p+1)} \left| \ln \left\| f - f_* \right\| \right| \left\| f - f_* \right\|^{p/(p+1)}, 0 < p \leq 2.$$

To find the proper regularization parameter from the set $L_{min}$ we consider so-called $Q$-curve method. $Q$-curve is a plot of the function $\log \psi_Q(\alpha)$ versus the corresponding modified residual $\log d(\alpha)$, where $d(\alpha) = \langle Au_{2,\alpha} - f, Au_\alpha - f \rangle$.