Abstracts of MMA2016, June 1–4, 2016, Tartu, Estonia

REGULARIZATION OF ILL-POSED PROBLEMS VIA REGULARIZATION OF THEIR DISCRETIZATION

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A stable solving of an ill-posed problem $Au = f$ formulated in abstract spaces X, Y can be achieved by using various regularization strategies. Many regularization methods are formulated just in these infinite dimensional spaces. In general, such regularized problems are numerically not feasible and discretization is required to treat related well-posed problems. Another approach consists in discretization of the ill-posed problem under consideration. As a result of discretization one obtains so called a discrete ill-posed problem, i.e. a family of finite dimensional ill-conditioned problems for which the condition number tends to infinity with the increasing dimension of discrete problem.

The subject of this presentation is an idea of combining the projection- and the Tikhonov regularization methods for an ill-posed lnear equation in Hilbert spaces. The Tikhonov method is applied to the discrete ill posed problem. It is a novelty of our approach that the discretization level is treated as the second parameter of regularization. So, we deal with two-parameter regularization of the main problem in infinite dimensional spaces.

An a priori choice of these parameters was consider in [1]. Now, we focus our attention on an a-posteriori parameter choice rule. In [2] a modified discrepancy principle is used for defining a discrepancy set $DS(\delta)$ of pairs (n, α) for any fixed data error bound δ, in such a way that the corresponding regularized solutions generated by $(n, \alpha) \in DS(\delta)$ approximates the exact one with the same order. Here n denotes the discretization level and α is the Tikhonov regularization parameter. In my talk a further analysis of the interaction between the Tikhonow regularization parameter and the discretization level belonging to $DS(\delta)$ will be presented for some projection methods. The convergence rate with respect to δ will be discussed for a regularized solution generated by $(n(\alpha), \alpha) \in DS(\delta)$ under the standard source conditions. Under certain assumptions, the optimal order of accuracy can be achieved.

In the quite recently published paper [3] the authors consider a similar approach for nonlinear ill-posed problem in Banach spaces.

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