

A DISCRETE LIMIT THEOREM FOR THE PERIODIC HURWITZ ZETA-FUNCTION. II

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Let $0 < \alpha \leq 1$ be a fixed parameter, and $\mathbf{a} = \{a_m : m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}\}$ be a periodic sequence of complex numbers. The periodic Hurwitz zeta-function $\zeta(s, \alpha; \mathbf{a})$, $s = \sigma + it$, is defined, for $\sigma > 1$, by the series

$$\zeta(s, \alpha; \mathbf{a}) = \sum_{m=0}^{\infty} \frac{a_m}{(m + \alpha)^s},$$

and by analytic continuation elsewhere. For the investigation of asymptotic behaviour of $\zeta(s, \alpha; \mathbf{a})$, the probabilistic approach can be applied. More precisely, limit theorems for weakly convergent probability measures can be obtained. Denote by $\mathcal{B}(\mathbb{C})$ the Borel σ -field of the complex plane \mathbb{C} . In [1], we obtained a limit theorem, as $N \rightarrow \infty$, with explicitly given limit measure for

$$\frac{1}{N+1} \# \{0 \leq k \leq N : \zeta(s + ikh, \alpha; \mathbf{a}) \in A\}, \quad A \in \mathcal{B}(\mathbb{C}),$$

provided that the set $\{(\log(m + \alpha) : m \in \mathbb{N}_0), \frac{\pi}{h}\}$, $h > 0$, is linearly independent over the field of rational numbers \mathbb{Q} . In the report, we consider the weak convergence of

$$\frac{1}{N+1} \# \{0 \leq k \leq N : \zeta(s + ik^\beta h, \alpha; \mathbf{a}) \in A\}, \quad A \in \mathcal{B}(\mathbb{C}),$$

where $h > 0$ and $0 < \beta < 1$ are fixed numbers. For this, the properties of uniformly distributed modulo 1 sequences of real numbers are applied.

REFERENCES

- [1] A. Rimkevičienė. A discrete limit theorem for the periodic Hurwitz zeta-function. *Liet. matem. rink. Proc. LMS, Ser. A*, **56**: 90–94, 2015.