Abstracts of MMA2016, June 1–4, 2016, Tartu, Estonia

SOLUTION OF QUADRATIC/LINEAR RATIONAL SPLINE HISTOPOLATION PROBLEM

EVELY KIRSIAED, PEETER OJA and **GUL WALI SHAH**

Institute of Mathematics and Statistics, University of Tartu

J. Liivi 2, 50409 Tartu, Estonia

E-mail: evely.kirsiaed@ut.ee, peeter.oja@ut.ee, gulwali@ut.ee

For given mesh $a = x_0 < x_1 < \ldots < x_n = b$ and histogram heights z_i, $i = 1, \ldots, n$, consider the problem of finding a C^2-smooth quadratic/linear rational histopolant satisfying

$$\int_{x_{i-1}}^{x_i} S(x)dx = z_i(x_i - x_{i-1}), \quad i = 1, \ldots, n. \tag{1}$$

Assume that the given histogram is strictly convex, i.e.,

$$D_i = (h_i + h_{i+1})z_{i-1} - (h_{i-1} + 2h_i + h_{i+1})z_i + (h_{i-1} + h_i)z_{i+1} > 0, \quad i = 2, \ldots, n-1,$$

where $h_i = x_i - x_{i-1}$, $i = 1, \ldots, n$. This is consistent with the fact that quadratic/linear rational splines are convex or concave everywhere.

Choose spline knots ξ_i, $i = 1, \ldots, n$, such that $\xi_1 = x_0$, $\xi_i \in (x_{i-1}, x_i)$, $i = 2, \ldots, n-1$, $\xi_n = x_n$, and present quadratic/linear rational spline via the second moments $M_i = S''(\xi_i)$, $i = 1, \ldots, n$, and integrals

$$\lambda_i = \int_{\xi_i}^{x_i} S(x)dx, \quad \rho_i = \int_{x_i}^{\xi_{i+1}} S(x)dx, \quad i = 1, \ldots, n-1,$$

i.e., on each interval $[\xi_i, \xi_{i+1}]$, $i = 1, \ldots, n-1$, we use four parameters $M_i, M_{i+1}, \lambda_i, \rho_i$.

To determine these parameters use histopolation conditions (1), smoothness conditions

$$S(\xi_i - 0) = S(\xi_i + 0), \quad S'(\xi_i - 0) = S'(\xi_i + 0), \quad i = 2, \ldots, n-1,$$

and two boundary conditions $S''(a) = \alpha$, $S''(b) = \beta$, yielding a quite complicated nonlinear system

$$\begin{cases} M_1 = \alpha, \\ \Phi_i(M_1, M_2, M_3, M_4) = D_2, \\ \Phi_i(M_{i-2}, M_{i-1}, M_i, M_{i+1}, M_{i+2}) = D_i, \quad i = 3, \ldots, n-2, \\ \Phi_i(M_{n-3}, M_{n-2}, M_{n-1}, M_n) = D_{n-1}, \\ M_n = \beta. \end{cases} \tag{2}$$

It is known that there are examples with strictly convex data where the solution does not exist [1]. We give some positive results where the proper choice of spline knots ξ_i ensures the existence of the solution. System (2) is analysed in details in case of any mesh and any placement of spline knots.

REFERENCES

[1] H. Hallik. *Rational Spline Histopolation (PhD thesis)*. Tartu University Press, 2015.