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STANDARD DIFFERENCE SCHEME IN THE PRESENCE OF PERTURBATIONS FOR A SINGULARLY PERTURBED CONVECTION-DIFFUSION ELLIPTIC EQUATION [1]

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Behavior of solutions for singularly perturbed problems (problems with a perturbation parameter ε, $\varepsilon \in (0,1]$, multiplying the highest-order derivatives) is much more complicated as compared to solutions of regular problems due to appearance of the boundary and/or the interior layers, which leads to necessity to develop special numerical methods convergent ε-uniformly in the maximum norm. Such methods have been developed only for sufficiently narrow classes of singularly perturbed problems (see, e.g., [1,2] and the bibliography therein). At the same time, numerical methods based on standard finite difference schemes (see., e.g., [3] and the references therein) are often used to solve problems with boundary layers. For convergence of solutions for such difference schemes, it is sufficient to use uniform meshes provided that grid step-size across the boundary layer is much smaller than ε [1,2]. As shown in [4], even in the case of an singularly perturbed ordinary convection-diffusion differential equation, standard schemes on uniform grids are not as ε-uniformly well-conditioned and ε-uniformly stable to the perturbation in the grid problem data, in particular, to computer perturbations; errors in the grid solution may exceed the solution itself.

In this talk, for a singularly perturbed elliptic convection-diffusion equation on a rectangle, an approach is considered to study stability of solutions for the standard difference scheme on a uniform grid in the presence of perturbations in the data of the grid problem and/or computer perturbations. We establish the conditions imposed on perturbations depending on ε, N_1, N_2, under which the perturbed grid solution converges as N_1, $N_2 \to \infty$; here N_1 and N_2 are numbers of intervals in the uniform grids in x_1 and x_2, respectively [5]. These conditions are already allow us to use the standard difference schemes on uniform grids for solving singularly perturbed problems to elliptic convection-diffusion equations in the presence of perturbations.

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