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A WEIGHTED UNIVERSALITY THEOREM FOR THE PERIODIC ZETA-FUNCTION

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We consider the periodic zeta-function $\zeta(s;\mathfrak{a})$, $s=\sigma+\mathrm{i}t$, defined, for $\sigma>1$, by the series

$$\zeta(s;\mathfrak{a})=\sum_{m=1}^{\infty}\frac{a_m}{m^s},$$

where $\mathfrak{a}=\{a_m:\ m\in\mathbb{N}\}$ is a periodic sequence of complex numbers with minimal period $q\in\mathbb{N}$. The function $\zeta(s;\mathfrak{a})$ is meromophically continuable to the whole complex plane.

The function $\zeta(s;\mathfrak{a})$, as the majority of zeta-functions, has the universality property on the approximation of analytic functions by shifts $\zeta(s+\mathrm{i}\tau;\mathfrak{a})$, $\tau\in\mathbb{R}$, however, its universality is rather complicated, it is known that not for all sequences $\mathfrak{a}$ the function $\zeta(s;\mathfrak{a})$ is universal.

We suppose that the sequence $\mathfrak{a}$ is multiplicative one, i.e., $a_{mn}=a_m a_n$ for all $m,n\in\mathbb{N}$, $(m,n)=1$. The firs result of such a kind has been obtained in [1]. The aim of this report is to present a weighted universality theorem for the function $\zeta(s;\mathfrak{a})$. Let $w(t)$ be a positive function of bounded variation on $[T_0,\infty)$, $T_0>0$, such that the variation $V_a^b w$ on $[a,b]$ satisfies the inequality $V_a^b w\le cw(a)$ with a certain $c>0$ for any subinterval $[a,b]\subset[T_0,\infty)$. Define

$$U=U(T,w)=\int_0^T w(t)\,\mathrm{d}\,t$$

and suppose that $U(T,w)\to\infty$ as $T\to\infty$. Let I_A stand for the indicator function of $A\subset\mathbb{R}$, $\mathcal{K}$ be the class of compact subsets of the strip $\{s\in\mathbb{C}:\ \frac{1}{2}<\sigma<1\}$ with connected complements, and let $H_0(K)$, $K\in\mathcal{K}$, denote the class of continuous non-vanishing functions on K which are analytic in the interior of K.

Theorem 1. *Suppose that the function $w(t)$ satisfies all above conditions, and that the sequence $\mathfrak{a}$ is multiplicative. Let $K\in\mathcal{K}$ and $f(s)\in H_0(K)$. Then, for every $\varepsilon>0$,*

$$\liminf_{T\to\infty}\frac{1}{U}\int_{T_0}^{T} w(\tau)I_{\left\{\tau:\sup_{s\in K}|\zeta(s+\mathrm{i}\tau;\mathfrak{a})-f(s)|<\varepsilon\right\}}\,\mathrm{d}\,\tau>0.$$

REFERENCES

[1] A. Laurinčikas and D. Šiaučiūnas. Remarks on the universality of the periodic zeta-functions. *Math. Notes*, **80** (4):532–538, 2006.