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DISCRETE UNIVERSALITY THEOREM FOR THE PERIODIC HURWITZ ZETA-FUNCTION

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Let $s = \sigma + it$ be a complex variable, $0 < \alpha \leq 1$ be a fixed parameter, and let $\mathfrak{a} = \{a_m : m = 0, 1, \dots\}$ be a periodic sequence of complex numbers. The periodic Hurwitz zeta-function $\zeta(s, \alpha; \mathfrak{a})$ is defined, for $\sigma > 1$, by the series

$$\zeta(s, \alpha; \mathfrak{a}) = \sum_{m=0}^{\infty} \frac{a_m}{(m+\alpha)^s},$$

and by analytic continuation elsewhere.

In the report, we consider the discrete universality of the function $\zeta(s, \alpha; \mathfrak{a})$. Let $h > 0$, $0 < \beta_1 < 1$ and $\beta_2 > 0$ be fixed numbers. We discuss the approximation of a wide class of analytic functions by shifts $\zeta(s + ihk^{\beta_1} \log^{\beta_2} k, \alpha; \mathfrak{a})$, $k = 2, 3, \dots$. For a precise statement of our result, we use the following notation. Let $\mathcal{K}$ be the class of compact subsets of the strip $\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\}$ with connected complements, and let $H(K)$, $K \in \mathcal{K}$, denote the class of continuous functions on K which are analytic in the interior of K. Moreover, let $L(\alpha) = \{\log(m + \alpha) : m = 0, 1, \dots\}$, and let $\#A$ denote the cardinality of the set A.

Theorem 1. *Suppose that the set $L(\alpha)$ is linearly independent over the field of rational numbers. Let $K \in \mathcal{K}$ and $f(s) \in H(K)$. Then, for every $\varepsilon > 0$,*

$$\liminf_{N \to \infty} \frac{1}{N-1} \# \left\{ 2 \leq k \leq N : \sup_{s \in K} \left| \zeta(s + ihk^{\beta_1} \log^{\beta_2} k, \alpha; \mathfrak{a}) - f(s) \right| < \varepsilon \right\} > 0.$$

For the proof of the theorem, a probabilistic method based on limit theorems for weakly convergent probability measures in the space of analytic functions as well as the properties of uniformly distributed modulo 1 sequences of real numbers are applied.