SPECTRUM CURVES OF DISKRETE STURM–LIOUVILLE PROBLEM WITH INTEGRAL CONDITION

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We investigate SLP with one classical boundary condition and another integral type NBC:

\[-u'' = \lambda u, \quad t \in (0, 1), \quad \lambda \in \mathbb{C}, \quad \lambda := \mathbb{C},
\]

\[u(0) = 0, \quad u(1) = \gamma \int_{\xi_1}^{\xi_2} u(t) \,dt,
\]

where NBC’s parameter \(\gamma \in \mathbb{R}\) and \(\xi \in S_\xi := \{(\xi_1, \xi_2) \in [0, 1]^2 : 0 \leq \xi_1 < \xi_2 \leq 1\}\).

We introduce a uniform grid \(\mathcal{J}^h = \{t_j = jh, j = 0, n; n \in \mathbb{N}, nh = 1\}\) in the interval \([0, 1]\). Also, we make an assumption, that \(\xi_1\) and \(\xi_2\) are coincident with grid points, i.e., \(\xi_1 = m_1h = m_1/n, \quad \xi_2 = m_2h = m_2/n, \quad m \in S_h := \{(m_1, m_2) : 0 \leq m_1 < m_2 \leq n\}\). We approximate differential SLP (1)–(2) by the Finite-Difference Scheme (discrete SLP):

\[\frac{U_{j-1} - 2U_j + U_{j+1}}{h^2} + \lambda U_j = 0, \quad j = 1, \ldots, n - 1, \quad U_0 = 0, \quad U_n = \gamma h \left( \frac{U_{m_1} + U_{m_2}}{2} + \sum_{k=m_1+1}^{m_2-1} U_k \right),
\]

where for approximation of the integral in NBC we use trapezoidal formula. We investigate discrete SLP and analyze how complex eigenvalues of this problem depend on the parameters of the integral NBC. Some results for the both SLP were presented in [1–5].

REFERENCES


\(^{1}\) The research was partially supported by the Research Council of Lithuania (grant No. MIP-047/2014).