

SPECTRUM CURVES OF DISKRETE STURM–LIOUVILLE PROBLEM WITH INTEGRAL CONDITION ¹

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We investigate SLP with one classical boundary condition and another integral type NBC:

$$-u'' = \lambda u, \quad t \in (0, 1), \quad \lambda \in \mathbb{C}_\lambda := \mathbb{C}, \quad (1)$$

$$u(0) = 0, \quad u(1) = \gamma \int_{\xi_1}^{\xi_2} u(t) dt, \quad (2)$$

where NBC's parameter $\gamma \in \mathbb{R}$ and $\boldsymbol{\xi} \in S_{\boldsymbol{\xi}} := \{(\xi_1, \xi_2) \in [0, 1]^2 : 0 \leq \xi_1 < \xi_2 \leq 1\}$.

We introduce a uniform grid $\bar{\omega}^h = \{t_j = jh, j = \overline{0, n}; n \in \mathbb{N}, nh = 1\}$ in the interval $[0, 1]$. Also, we make an assumption, that ξ_1 and ξ_2 are coincident with grid points, i.e., $\xi_1 = m_1h = m_1/n$, $\xi_2 = m_2h = m_2/n$, $\mathbf{m} \in S_{\boldsymbol{\xi}}^h := \{(m_1, m_2) : 0 \leq m_1 < m_2 \leq n\}$. We approximate differential SLP (1)–(2) by the *Finite-Difference Scheme* (discrete SLP):

$$\frac{U_{j-1} - 2U_j + U_{j+1}}{h^2} + \lambda U_j = 0, \quad j = 1, \dots, n-1, \quad U_0 = 0, \quad U_n = \gamma h \left(\frac{U_{m_1} + U_{m_2}}{2} + \sum_{k=m_1+1}^{m_2-1} U_k \right),$$

where for approximation of the integral in NBC we use trapezoidal formula. We investigate discrete SLP and analyze how complex eigenvalues of this problem depend on the parameters of the integral NBC. Some results for the both SLP were presented in [1–5].

REFERENCES

- [1] R. Čiupaila, Ž. Jesevičiūtė, M. Sapagovas. On the eigenvalue problem for one-dimensional differential operator with nonlocal integral condition. *Nonlinear Anal. Model. Control*, **9**(2):109–116, 2004.
- [2] S. Pečiulytė, O. Štikonienė, A. Štikonas. Sturm–Liouville problem for stationary differential operator with nonlocal integral boundary condition. *Math. Model. Anal.*, **10**(4):377–392, 2005.
- [3] A. Skučaitė, K. Skučaitė-Bingelė, S. Pečiulytė, A. Štikonas. Investigation of the spectrum for the Sturm–Liouville problem with one integral boundary condition. *Nonlinear Anal. Model. Control*, **15**(4):501–512, 2010.
- [4] A. Skučaitė, K. Skučaitė-Bingelė, S. Pečiulytė, A. Štikonas. Investigation of the Sturm–Liouville problems with integral boundary condition. *Liet. matem. rink. LMD darbai*, **52**:297–302, 2011.
- [5] A. Skučaitė, A. Štikonas. Spectrum curves for Sturm–Liouville Problem with Integral Boundary Condition. *Math. Model. Anal.*, **20**(6):802–818, 2015. <http://dx.doi.org/10.3846/13926292.2015.1116470>

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