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ANALYTIC SOLUTIONS OF SINGULAR FRACTIONAL DIFFERENTIAL EQUATIONS

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Let J^ν be the Riemann-Liouville fractional integral operator. Define fractional differential operator D_0^ν, $\nu \geq 0$ on $J^\nu C[0,T]$ by $D_0^\nu = (J^\nu)^{-1}$ and the multiplication operator M_α for $\alpha \in \mathbb{R}$ by $(M_\alpha u)(t) = t^\alpha u(t)$. We consider singular fractional differential equations

$$(D_0^\alpha M_\alpha u)(t) = \sum_{k=1}^{l} a_k(t)(D_0^{\alpha_k} M_{\alpha_k} u)(t) + f(t), \quad 0 < t \leq T, \tag{1}$$

where $\alpha > \alpha_k \geq 0$, functions a_k, f are sufficiently regular and u is unknown. These equations are considered in [3] in spaces $C^m[0,T]$. We are interested under what conditions these equations admit analytic solutions in some region around $[0,T]$.

We make a change of variables $v = D_0^\alpha M_\alpha u$ and reduce the equation to a cordial integral equation with respect to v. Then we can use the results of [2, 4, 5].

Let D be a bounded domain in the complex plane containing $[0,T]$, which is star-shaped with respect to 0. Let $\mathcal{A}(D)$ be the space of functions analytic in D and continuous on $\overline{D}$ with the norm $\|v\|_{\mathcal{A}(D)} = \max_{t\in\overline{D}} |v(t)|$.

The main result for equation (1) is the following [1].

THEOREM 1. *Let $\alpha > \alpha_k \geq 0$, $a_k \in \mathcal{A}(D)$, $k = 1,\dots,l$ and $f \in \mathcal{A}(D)$ be given. Assume that* $\sum_{k=1}^{l} a_k(0)\frac{\Gamma(\alpha_k+n+1)}{\Gamma(\alpha+n+1)} \neq 1, \quad n = 0,1,\dots.$ *Then equation* (1) *has a unique solution* $u \in \mathcal{A}(D)$.

We consider the polynomial collocation method for solving (1) with Chebyshev points as the collocation points and show that this method converges exponentially in the number of unknowns.

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