

## ON CONVERGENCE AND TWO-SCALE ESTIMATES FOR CERTAIN FINITE-DIMENSIONAL OPERATORS

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For justifying numerical solution of multidimensional singular integral equation

$$(Au)(x) \equiv a(x)u(x) + v.p. \int_{\mathbb{R}^m} K(x, x-y)u(y)dy = v(x), \quad x \in \mathbb{R}^m, \quad (1)$$

with variable Calderon–Zygmund kernel  $K(x, y)$  [1] we try to apply methods of the theory of multidimensional singular integral operators, and first to study operators and equations with simplest kernel  $K(x, y) \equiv K(y)$  and  $a(x) \equiv a$ . Main goal of the studying is to obtain solvability (or Fredholmness) conditions of approximating equation from starting conditions of the given equation (1).

We introduce a discrete analogue of an integral operator in the (1) as the series

$$(A_d u_d)(\tilde{x}) \equiv a(\tilde{x})u_d(\tilde{x}) + \sum_{\tilde{y} \in h\mathbb{Z}^m} K_d(\tilde{x}, \tilde{x} - \tilde{y})u_d(\tilde{y})h^m, \quad \tilde{x} \in h\mathbb{Z}^m, \quad h > 0, \quad (2)$$

where by definition  $K(x, 0) \equiv 0, \forall x \in \mathbb{R}^m$ , and partial sums are taken by cubes.

It was shown earlier [3–6] that in simplest cases the operator (2) in general inherits spectral properties of the operator (1). Key role in this studying played the theory of periodic Riemann boundary value problem [7]. Some comparison estimates for operators (1) and (2) were given in [2, 4]. Next step in author's studies in this direction is constructing finite approximations for the operator (2) and obtaining corresponding comparison estimates for the operator (1) and its finite-dimensional analogue. This report based on a joint work with A. V. Vasilyev.

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