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NUMERICAL SOLUTION OF NONLINEAR FRACTIONAL BOUNDARY VALUE PROBLEMS

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We consider a boundary value problem for a nonlinear fractional differential equation of the form

$$(D_*^\alpha y)(t) = f(t, y(t)), \quad 0 \le t \le b, \tag{1}$$

with conditions

$$\sum_{j=0}^{n_0} \beta_{ij0}\, y^{(j)}(0) + \sum_{k=1}^{l} \sum_{j=0}^{n_1} \beta_{ijk}\, y^{(j)}(b_k) = \gamma_i\,, \quad i = 0, \ldots, n-1\,,\ n := \lceil \alpha \rceil, \tag{2}$$

where $\beta_{ij0}, \beta_{ijk}, \gamma_i \in \mathbb{R} := (-\infty, \infty)$, $\lceil \alpha \rceil$ is the smallest integer greater or equal to $\alpha \in \mathbb{R}$,

$$n-1 < \alpha < n\,, \quad 0 < b_1 < \cdots < b_l \le b,$$
$$l, n \in \mathbb{N} := \{1, 2, \ldots\}, \quad n_0, n_1 \in \mathbb{N}_0 := \{0\} \cup \mathbb{N}, \quad n_0 < n,\ n_1 < n,$$

$f : [0, b] \times \mathbb{R} \to \mathbb{R}$ is a given continuous function, and $D_*^\alpha y$ is the Caputo derivative (see e.g. [1]) of order $\alpha > 0$ of an unknown function y.

We construct a class of high order methods for the numerical solution of (1)-(2). Using an integral equation reformulation of the underlying problem we first study the smoothness of the exact solution and regularize the solution by a suitable smoothing transformation. After that we solve the transformed equation by a piecewise polynomial collocation method on a non-uniform or uniform grid. The final step of our method is based on a conversion of the obtained spline solution into (typically non-polynomial) approximate solution to (1)-(2). Optimal global convergence estimates are derived and a superconvergence result for a special choice of collocation parameters is established. The theoretical results are tested by some numerical examples.

REFERENCES

[1] K. Diethelm. *The Analysis of Fractional Differential Equations.* Lecture Notes in Mathematics, vol. 2004, Springer, Berlin, 2010.