

ACCELERATION DUE TO GRAVITY IN *STAR WARS*

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In *Star Wars*, every planet has the same acceleration due to gravity as the Earth. Why?

How does the mass of a planet depend on its radius in that case? How to realise this relation in practice? ¹

The force of gravity between two spherical bodies is

$$(1) \quad F = \frac{Gm_1m_2}{r^2},$$

where G is the gravitational constant, m_1 and m_2 are masses of the bodies, and r is the distance between them.

We are interested of the acceleration due to gravity g of a planet, whose mass is M and radius is R , on a small body with mass m .

Newton's second law says that $F = ma$.

The force of gravity between the body on the surface of the planet is on one hand

$$(2) \quad F = \frac{GMm}{R^2},$$

on the other hand *defined* as

$$(3) \quad F = mg.$$

We find that

$$(4) \quad g = \frac{GM}{R^2}.$$

If g is constant independent of the mass of the planet, then we have

$$(5) \quad \frac{M}{R^2} = \frac{g}{G} = \text{const},$$

that is, the mass of the planet is proportional to its radius squared!

It is quite an odd result: M rather ought to be proportional to R^3 , but even that assumes that the planet is of uniform density.

If mass is proportional to the square of radius, i.e. to the surface area of the planet, it looks like the whole mass of the planet is near its surface: the planet is hollow!

How dense must the spherical shell be? The radius of the Earth is about $R_E = 6400$ km and its mass $M_E = \times 10^{24}$ kg. Its surface area is then $S_E = 4\pi R_E^2 = 5 \times 10^8$ km². The mass per one square kilometre is 10^{16} kg, per one square metre: 10^{10} kg.

If the thickness of the shell of an Earth-sized planet were 1 km, its density would be about 10^7 kg/m³. The average density of the Earth is about

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5500 kg/m³, almost ten thousand times less; the density of white dwarf star matter is only 100 times larger!

The shell, even if it could be formed, would collapse under its own weight.

There is a much more realistic way to hold the acceleration due to gravity constant: one has to change the ratio of radii of the high density iron-nickel core and the relatively less dense shell of a quite ordinary planet.

Let us assume that the planet with radius R has a core with radius R_c (of course, $R_c < R$). Let the density of the core be ρ_c and the density of the shell ρ_s (of course, $\rho_s < \rho_c$).

Then the mass M of the planet is the sum of the masses of its core and shell:

$$(6) \quad M = V_c \rho_c + V_s \rho_s = \frac{4}{3} \pi R_c^3 \rho_c + \left(\frac{4}{3} \pi R^3 - \frac{4}{3} \pi R_c^3 \right) \rho_s = \frac{4}{3} \pi R^3 [\rho_s + \chi^3 (\rho_c - \rho_s)],$$

where we have defined the ratio of radii as $\chi = R_c/R$.

We know that for a *Star Wars* planet $M/R^2 = g/G$. Thus

$$(7) \quad \frac{g}{G} = \frac{4}{3} \pi R [\rho_s + \chi^3 (\rho_c - \rho_s)],$$

from where the ratio of radii of the core and the planet (shell) is

$$(8) \quad \chi(R) = \sqrt[3]{\frac{-3g/G + 4\pi R \rho_s}{4\pi R (\rho_s - \rho_c)}}.$$

Let us plot the relationship for $\rho_s = 4000 \text{ kg/m}^3$ and $\rho_c = 20000 \text{ kg/m}^3$.

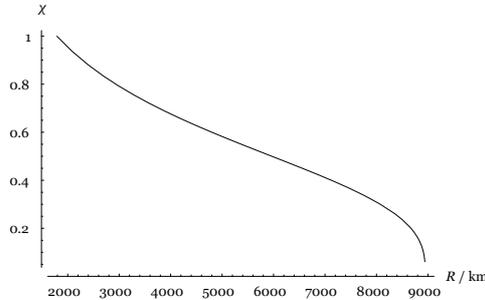


FIGURE 1. The ratio χ of the core and planet a function of the radius R of the planet, if the acceleration to to gravity on the surface of the planet is held at g .

From the figure we see that with the help of this trick, we can make planets whose radius is as small as 1800 km or as large as 8800 km. (The actual range is smaller, as it is a rough estimation.)

To further increase the radius of a planet, while keeping g constant, an ice shell may be added. The density of ice is about 1000 kg/m³ (larger under a great pressure). In this way, the 8800 km limit can be passed.