Moon-Moseri graafid

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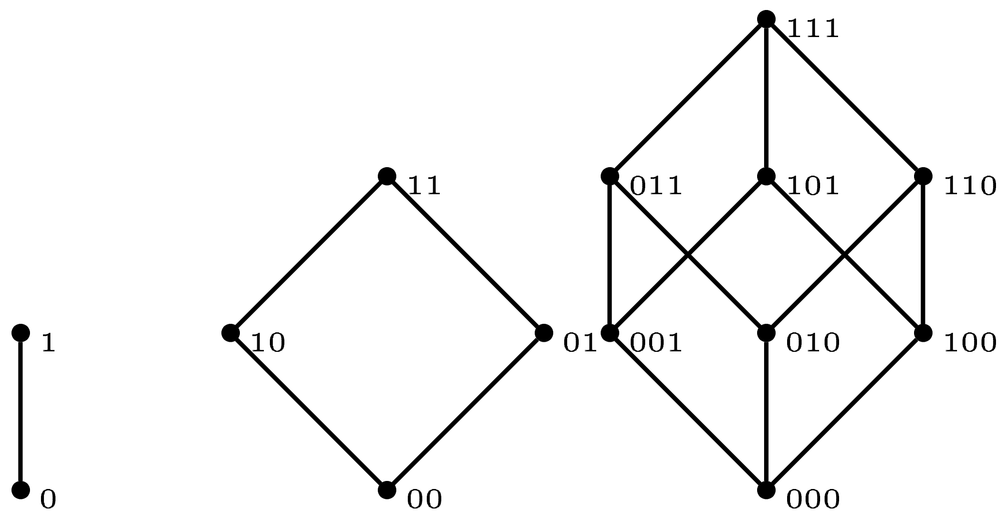
Partial order $\prec$ on B^n is defined as:

$$\alpha \prec \beta \equiv \exists i[\alpha_i < \beta_i \;\&\; \forall j((j \neq i) \rightarrow (\alpha_j = \beta_j))],$$

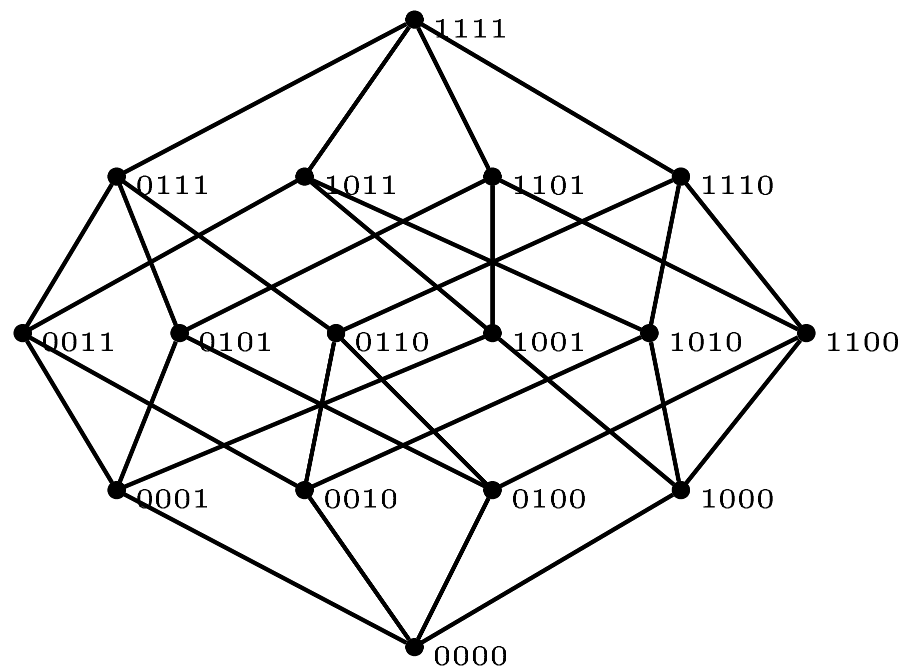
where $\alpha = \alpha_1 \ldots \alpha_n \in \{0,1\}^n$ $\beta = \beta_1 \ldots \beta_n \in \{0,1\}^n$.

By $\prec^+$ we denote the transitive closure of the relation $\prec$.

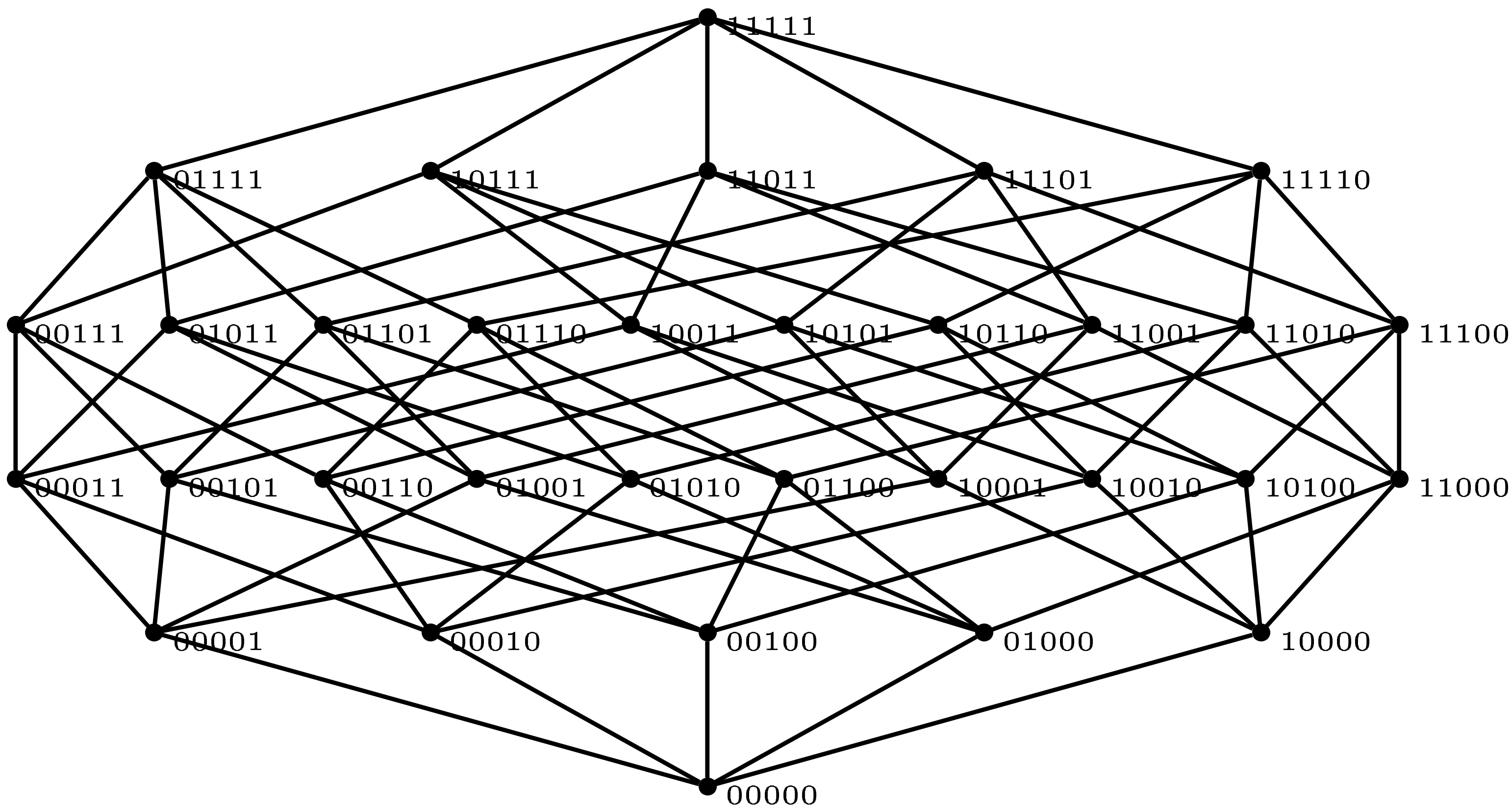


BL(1) BL(2) BL(3)

Boolean lattices with 1, 2 and 3 generators.



Boolean lattice BL(4)



Boolean lattice BL(5)

Function $f \in \mathcal{F}^n$ is *monotone*, if

$(\alpha_1, \ldots, \alpha_n) \prec^+ (\beta_1, \ldots, \beta_n) \to f(\alpha_1, \ldots, \alpha_n) \leq f(\beta_1, \ldots, \beta_n)$

Function $f \in \mathcal{F}^n$ is *antimonotone*, if

$(\alpha_1, \ldots, \alpha_n) \prec^+ (\beta_1, \ldots, \beta_n) \to f(\alpha_1, \ldots, \alpha_n) \geq f(\beta_1, \ldots, \beta_n)$

Function $f \in \mathcal{F}^n$ is *antichain*, if

$f(\alpha) = f(\beta)$ implies α and β are not comparable by $\prec^+$

Let $f \in \mathcal{F}^n$ be an antimonotone function, f^1 the antichain of its maximal ones and f^0 the antichain of its minimal zeroes.

Theorem. Minimal CNF for f is

$$\bigwedge_{\alpha \in f^0} \left(\bigvee_{\alpha_i = 1} \overline{x_i} \right).$$

Theorem. Minimal DNF for f is

$$\bigvee_{\alpha \in f^1} \left(\bigwedge_{\alpha_i = 0} \overline{x_i} \right).$$

Let $G = (V, E)$ be a graph. We define a CNF

$$f_G = \bigwedge_{\{i,j\} \in co-E} (\overline{x_i} \vee \overline{x_j})$$

Theorem. Let G be a connected graph. $f_G(\alpha) = 1$ if and only if α is a characteristic vector of some complete subgraph of G.

Corollary Antichain of maximal ones of f_G consists of characteristic vectors of cliques of G.

Let $G_{l,k}$ be a graph, which consists of k isolated l-vertex complete graphs K_l.

Moon- Moser graph $M_{l,k} = co - G_{l,k}$.

Theorem. The graph $M_{l,k}$ has l^k cliques, each of k vertices.

Theorem. Maximal number of cliques in a graph with n nodes, where $n \equiv 0 (mod 3)$ is $3^{n/3}$.

Theorem. Minimal CNF for $f_{M_{3,k}}$ has $n = 3 \cdot k$ clauses and a minimal DNF has $3^{n/3} = 2^{0.528333 \cdot n}$ terms.

Let $\mathcal{A}(x_1, \ldots, x_n)$ be a propositional formula. We denote by $\mathcal{A}_{x_i}$ ($\mathcal{A}_{\overline{x_i}}$) a formula which we receive from $\mathcal{A}$ by substituting 1 (0) instead of every occurence of variable x and simplifying the result according to the rules of propositional calculus.

Theorem.(Shannon expansion theorem.) Let $\mathcal{A}$ be a propositional formula and x a variable. Then

$$\mathcal{A} = x \& \mathcal{A}_x \vee \overline{x} \& \mathcal{A}_{\overline{x}}.$$

Algorithm #SAT

function $\#DP(F$: **set of clauses**, l : **integer**) : **integer**

begin $\#DP$

1. **if** $F = \emptyset$ **then return**(2^l);

2. **if** $\Box \in F$ **then return**(0);

3. **if** F contains unit clause, say a, **then return** $(\#DP(F_a, l-1))$;

4. Choose a literal a; **return** $(\#DP(F_a, l-1) + \#DP(F_{\bar{a}}, l-1))$.

end $\#DP$

Teoreem. $\#DP(F, n)$ on funktsiooni F lahendite arv.

Theorem. If CNF $F = F_1 \& F_2$, where F_1 and F_2 does not have common variables, then $\#F = \#F_1 \cdot \#F_2$.

function $\#DP(F$: **set of clauses**, l: **integer**) : **integer**

begin $\#DP$

1. **if** $F = \emptyset$ **then return**(2^l);

2. **if** $\Box \in F$ **then return**(0);

3. **if** F contains unit clause, say a, **then return** $(\#DP(F_a, l-1))$;

4. **if** $F = F_1(z_1, \ldots, z_k) \& F_2(y_1, \ldots, y_m)$, where F_1 and F_2 does not have common variables, **then** **return**$(2^{l-k-m} \cdot \#DP(F_1, k) \cdot \#DP(F_2, m))$.

5. Choose a literal a; **return** $(\#DP(F_a, l-1) + \#DP(F_{\bar{a}}, l-1))$.

end $\#DP$